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# REVIEWS

**Stochastic Tools in Turbulence.** By JOHN L. LUMLEY. Academic Press, 1970. 205 pp. £5.60.

**Turbulence Phenomena.** By J. T. DAVIES. Academic Press, 1972. 412 pp. $19.50.

The word 'turbulence' means different things to different investigators, and two opposite extremes are represented by the two books under review. The first, by Lumley, is devoted to an account of certain mathematical techniques available for the study of random functions. The second, by Davies, is an account of certain turbulence phenomena as they are observed, and of semi-empirical theories by which some order can be extracted from the chaos of observations. There could scarcely be a greater contrast in the philosophy or general style of approach to the subject of turbulence; and yet the amazing thing is that both types of book have a part to play in the business of understanding turbulence and disseminating results to students.

There are four chapters in Professor Lumley's book, of which the first three are devoted to the theory of random functions of a single variable, and the fourth to a consideration of properties of a random scalar or vector function of two or three space variables. There are also substantial appendices defining and discussing the basic mathematical techniques used in the main text (Fourier transforms, generalized functions and tensor invariant theory). Much of the material of the first three chapters is to be found in standard texts on probability theory (e.g. Feller's *Introduction to Probability Theory*); and much of the material of the fourth chapter in standard texts on turbulence (e.g. in chapters 1–3 of Batchelor's *Homogeneous Turbulence*); the novelty of Lumley's treatment lies in the degree of mathematical abstraction that he brings to bear on the topics considered – an abstraction that may unfortunately make the book quite inaccessible to many turbulence researchers – and the careful discussion that he devotes to a number of fine points that most others involved in turbulence are content either to accept or to ignore. Typical among the topics considered are (i) the definition of the time average for stationary processes and the conditions under which this is (in some sense) equal to the ensemble average; (ii) errors involved in obtaining moments and other statistical properties of a random function from a finite-interval sample; (iii) conditions under which the integral of a stationary random function has asymptotically Gaussian properties; (iv) the probability of the occurrence of a zero of a random function. The problem of inferring typical structures ('characteristic eddies') from knowledge of the two-time (or two-point) statistics is also considered at some length. This topic, of particular interest to experimentalists attempting to interpret measured spectra in terms of eddy structures, is well chosen; but as I read Professor Lumley's abstract and erudite argument, I yearned for the relief of a number of examples (starting with explicit correlations or spectra) to indicate in detail just how the technique advocated may be applied in practical contexts; but these, alas, were not provided.

It will be evident from the brief description given above that Lumley restricts attention in his book to purely kinematic aspects of turbulence, i.e. to problems associated with the instantaneous description of a random scalar or vector field. The *evolution* of the statistical properties of these fields, as governed by Navier–Stokes or other equations, is not considered. The title of the book is perhaps a little misleading: turbulence divorced from the Navier–Stokes equations is hardly turbulence. One cannot blame Professor Lumley for failing to apply his stochastic tools to solve the central nonlinear problems of turbulence – the available tools are probably woefully inadequate for this purpose; but one may perhaps accuse him of misleading the unwary or uninformed reader through his choice of title; such a reader may believe that if he masters the book he may be in a position to attack these central nonlinear problems of turbulence, but he will be disappointed: he will find himself metaphorically armed with some finely sharpened surgeon's scalpels and faced with the felling of an oak tree!

Professor Davies's book is aimed primarily at undergraduate chemical engineers. As such, it necessarily makes minimal appeal to sophisticated mathematics. Certainly no use is made of the stochastic tools advocated by Lumley! The character of the book lies in the range and interest of the problems considered. These reflect particularly Professor Davies's own interests in heat and mass transfer near liquid interfaces, where the adjacent phase may be either solid, liquid, or gas. Heat or suspended matter (e.g. salt) can diffuse across such an interface by molecular processes. The rate of transfer is greatly accelerated by the presence of turbulence on either or both sides of the interface. A vast range of experimental results is now available and Professor Davies provides an admirable description of the observed phenomena, illustrated by some excellent photographs. His analysis of the phenomena is restricted to a 'mixing-length' approach, supplemented by rules of thumb that lead to satisfactory or semi-plausible agreement between 'theory' and experiment.

Only the first chapter of the book "Velocities and stresses in turbulent flows" is concerned with momentum (as opposed to heat and mass) transfer, and it is introductory in character. I found the treatment of some basic topics in this chapter rather confusing. For example in deriving the logarithmic law of the wall, the author first expresses the shear stress (following Prandtl) in terms of the mixing length, then uses an empirical formula for the mixing length, and then integrates to obtain the mean velocity profile. It seems to be contrary to the spirit of modern fluid mechanics to make the derivation of a basic law (the logarithmic law) depend on an elusive and ill-defined concept (the mixing-length); the law can be derived (following Millikan) on pure dimensional analysis, and this is surely the best way to do it. A further example of obscure, or rather misleading, treatment is to be found later in the same chapter under the heading "Decay of isotropic intensity with time". The author makes the statement "the energy dissipated by the unit volume of the turbulent system (downstream of a grid) is dissipated ultimately by viscous flow, so that a first power dependence on viscosity is to be expected". This statement is quite wrong; on the universally accepted picture of high Reynolds number turbulence, the rate of energy dissipation per unit volume is at most weakly dependent on viscosity (and totally independent of viscosity if Kolmogorov's theory is correct).

Among the topics considered in subsequent chapters are (i) the mixing of two liquids in stirred tanks and in submerged jets, (ii) the deposition of solid particles in a turbulent stream on a solid boundary, (iii) the deformation of the free surface of a liquid due to local turbulence, (iv) turbulent flow in falling liquid films, (v) the effects of surface-active agents on transfer processes at liquid interfaces, (vi) turbulent drag reduction using polymer additives, (vii) heat transfer from drops falling through another liquid at high Reynolds number, (viii) spontaneous interfacial turbulence due to the creation of random variations in surface tension under transfer of surface-active solutes, (ix) the 'upward drilling' of bubbles into a melting solid, (x) the dispersion and coalescence of drops of one liquid due to turbulence in the surrounding liquid. The physical discussion of these phenomena is generally illuminating; and Professor Davies succeeds in conveying some of the excitement of current research work in these areas. The analysis of the problems, as presented, is however rudimentary; the book provides evidence of a wealth of problems of practical importance which invite attack using more detailed models for local fluid dynamic behaviour coupled with statistical methods to derive the mean properties of the systems studied.

In summary, there is little doubt that Professor Davies's book will provide stimulating reading for undergraduate chemical engineers, and a thought-provoking collection of phenomena for teachers and researchers alike.

H. K. Moffatt